

# CONDUCTORS, CAPACITORS, DIELECTRICS

## 1. ELECTROSTATICS WITH CONDUCTORS

In a steady state there is no electric field inside a conductor. Because electric charges move freely through a conductor, they quickly reposition themselves to cancel any electric field there. According to Gauss's law, if the electric field,  $\mathbf{E}$ , inside a conductor is zero, then the charge,  $Q$ , enclosed by any surface,  $\mathbf{A}$ , inside the conductor is also zero:

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = Q = 0$$

As a result, any excess charge must reside on the surface of the conductor. Therefore, the surface of a conductor is an equipotential, and the electric potential,  $V$ , must be uniform.

Because the electric field is perpendicular to a Gaussian surface, the magnitude of the electric flux through the surface is the product of the magnitude of the electric field,  $E$ , and the area of the surface,  $A$ :

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0} = EA$$

If the surface charge density is  $\sigma$ , then the magnitude of the electric field is given by:

$$E = \frac{Q}{A\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

## 2. CAPACITORS

**Capacitors** are devices used in electric circuits to store electric charge and energy and consist of two conductors known as **plates**. The potential difference,  $V$ , between the two plates is related to the charge,  $Q$ , on the plates by a proportionality constant,  $C$ , known as the **capacitance**:

$$Q = CV$$

The capacitance is a measure of the capacitor's ability to store charge. The unit of capacitance is the **farad** and is equivalent to one coulomb per volt.

A **parallel plate capacitor** consists of two parallel plates of area  $A$ , separated by a distance  $d$ . The voltage difference between the plates is given by:

$$V = Ed$$

The electric field,  $\mathbf{E}$ , between the plates can be related to the charge,  $Q$ , on the plates using Gauss's law:

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = \epsilon_0 EA = Q$$

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Therefore, the capacitance of a parallel plate capacitor is proportional to the plate area and the separation distance:

$$C = \epsilon_0 \frac{A}{d}$$

The plates of a **cylindrical capacitor** of length  $L$  consist of a cylindrical conductor of radius  $a$ , nested within a larger, hollow cylindrical conductor of radius  $b$ . For a cylindrical Gaussian surface of radius  $r$  between the plates, the electric field is perpendicular to the surface and constant in magnitude. Therefore, the relationship between the electric field and the charge on the plates is given by:

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = 2\epsilon_0 \pi L r E = Q$$

The voltage difference between the capacitor plates is obtained by integrating the electric field between the capacitor plates:

$$V = \int_a^b E dr = \frac{Q}{2\pi\epsilon_0 L} \ln(b/a)$$

The capacitance of the cylindrical capacitor is then given by:

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

**Spherical capacitor** plates consist of a spherical conductor of radius  $a$ , nested inside a spherical shell conductor of radius  $b$ . For a spherical surface of radius  $r$ , positioned between the plates, the electric field lines are straight and directed radially. The relationship between the electric field and the charge on the plates is given by:

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = 4\pi\epsilon_0 r^2 E = Q$$

The voltage difference between the capacitor plates is obtained by integrating the electric field between the capacitor plates:

$$V = \int_a^b E dr = \frac{Q}{4\pi\epsilon_0} \left( \frac{b-a}{ab} \right)$$

and the capacitance of a spherical capacitor is given by:

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_0 ab}{b-a}$$

Capacitors store energy as well as charge. The energy,  $U$ , stored by a capacitor of capacitance  $C$  that is charged to a voltage  $V$ , is given by:

$$U = \int_0^Q V dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

**3. DIELECTRICS**

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A **dielectric** is an insulating material placed between the plates in capacitors to keep the plates from coming in contact with each other. By using dielectrics, capacitors can be 'rolled' into a more compact form. The dielectric also increases the energy storage capacity of the capacitor, because work is done in aligning molecular **dipoles** in the dielectric material with the electric field in response to the charge on the capacitor plates. The dipole alignment gives rise to an opposing electric field that cancels some of the field between the plates. Therefore, the voltage across the plates is reduced and is given by:

$$V' = \frac{V}{\kappa}$$

where  $V$  is the voltage across the plates in the absence of a dielectric, and  $\kappa$  is a unitless constant known as the **dielectric constant**. Because the voltage of the capacitor decreases while the charge remains the same, the capacitance increases and is given by:

$$C' = \frac{Q}{V'} = \frac{\kappa Q}{V} = \kappa C$$

The capacitance of a parallel plate capacitor with a dielectric material between its plates, then, is given by:

$$C' = \frac{\kappa \epsilon_0 A}{d}$$

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